1 Units and Measurement

Launching a rocket into space is full of complexities. Engineers are required to have knowledge of aerodynamics, density, resistance, chemical energy, radiation and much more. Such complexities are made simple with use of fundamental units such as length, mass and time. Precise calculations and minimal errors are key for a successful launch and one minor mistake can lead to a fatal crash. Thus, study of units and measurement is an important branch of physics.

Topic Notes

- *Concepts of Units and Their Applications*

CONCEPTS OF UNITS AND THEIR APPLICATIONS

1

TOPIC 1

UNITS

An arbitrarily chosen standard to measure an amount of physical quantity which is easily reproducible and internationally accepted is known as unit. If the length of a metal rod is measured in cm, the unit of length is cm.

Physical quantity = Numerical Part × Unit.

A System of Units is made up of both fundamental and derived units.

Units of Physical Quantities

All the quantities which can be measured directly or indirectly and in terms of which, laws of physics are described are called Physical quantities.

There are two types of units: Fundamental units and Derived units.

Those quantities which are independent of other physical quantities and are not usually defined in terms of their physical quantities are called fundamental quantities and their units are called fundamental units or base units.

There are seven fundamental quantities and two supplementary units.

Fundamental quantities and their SI units			
S. No.	**Name**	**Base quantity**	**Symbol**
1.	Length	Meter	m
2.	Mass	Kilogram	kg
3.	Time	Second	s
4.	Electric Current	Ampere	A
5.	Thermodynamic Temperature	Kelvin	K
6.	Amount of Substance	Mole	mol
7.	Luminous Intensity	Candela	cd
Supplementary Units			
1.	Plane angle	Radian	rad
2.	Solid angle	steradian	Sr

Important

➥ *One radian is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of the circle. One steradian is defined as the solid angle subtended at the centre of a sphere by a surface of the sphere equal in area to that of a square, having each side equal to the radius of the sphere.*

Those quantities, which can be derived from two or more than two fundamental physical quantities are called derived quantities. Velocity, acceleration, linear momentum, etc. The units of physical quantity like density, momentum, acceleration, force, work, power, energy, pressure etc., can be expressed in terms of fundamental units. Their units are derived units.

System of Units

It is a complete set of fundamental and derived units. We have four types of systems of units. Generally, a system is named in terms of fundamental units on which it is based.

(1) **M.K.S. system:** In this system length, mass and time are taken as fundamental quantities.

The main drawback of all the above systems is that they are confined to mechanics only. All the physical quantities appearing in physics cannot be described by these systems.

(2) **S.I. system:** The Système Internationale d' Unites (French for International System of Units) or SI was introduced in 1971 by the General Conference on Weights and Measures. It is also called a rationalised M.K.S. system because it is made by modifying the M.K.S. system. It is nothing but an extended M.K.S. system. It is a comprehensive system.

(3) **C.G.S. system:** It is a Gaussian system. In this also length, mass and time are taken as fundamental quantities. The disadvantage of the C.G.S. system is that many derived units in this system become unnecessarily small.

(4) **F.P.S. system:** In this also length, mass and time are taken as fundamental quantities. It is a British Engineering system. F.P.S. system is not used much nowadays because of inconvenient multiples and submultiples.

Types of Physical Quantity	Physical Quantity	MKS (originated in France)	CGS (originated in France)	FPS (originated in Britain)
Fundamental	Length	m	cm	Ft
	Mass	kg	g	lb
	Time	sec	sec	sec
Derived	Force	N (Newton)	D (dyne)	Pdl (poundal)
	Work or Energy	J (Joule)	erg	ft- poundal
	Power	W (Watt)	erg/s	ft- poundal/s

Order of Magnitude

Order of magnitude of a quantity is the power of 10 required to represent that quantity. This power is determined after rounding off the value of the quantity properly. For rounding off, the last digit is simply ignored if it is less than 5 and is increased by one if it is 5 or more than 5.

When a number is divided by 10^x (where x is the order of the number) the result will always lie between 0.5 and 5 *i.e.*, $0.5 \le \frac{N}{10^x} < 5$.

Order of magnitude of some values can be determined as follows:

(1) $49 = 4.9 \times 10^1 \equiv 10^1$

Order of magnitude = 1

(2) $51 = 5.1 \times 10^1 \equiv 10^2$

$\therefore$ Order of magnitude = 2

(3) $0.0049 = 4.9 \times 10^{-2} \equiv 10^{-2}$

$\therefore$ Order of magnitude = –2

(4) $0.0050 = 5.0 \times 10^{-2} \equiv 10^{-1}$

$\therefore$ Order of magnitude = –1

(5) $0.0051 = 5.1 \times 10^{-2} \equiv 10^{-1}$

$\therefore$ Order of magnitude = –1

Example 1.1: Give the order of the following:

(A) 1

(B) 499

(C) 501

(D) 1 Å (10^{-10}m)

(E) Gravitational constant (6.67×10^{-11} N-m^2/kg^2)

(F) Planck's constant (6.63×10^{-34} m^{2}kg/s)

(G) Radius of H-atom (5.29×10^{-11} m)

(H) Mass of earth (5.98×10^{24} kg)

(I) 1 AU (1.496×10^{11} m)

(J) Speed of light (3.00×10^8 m/s)

(K) Avogadro constant (6.02×10^{23} mol^{-1})

(L) Charge of electron (1.60×10^{-19} C)

(M) Atmospheric pressure (1.01×10^5 Pa)

(N) Mean radius of earth (6.37×10^6 m)

Ans. (A) 0

(B) 2

Explanation: $499 = 4.9 \times 10^2 \equiv 10^2$

(C) 3

Explanation: $501 = 5.01 \times 10^2 \equiv 10^3$

(D) –10

(E) –10

(F) –33

(G) –10

(H) 25

(I) 11

(J) 8

(K) $24 = 6.0 \times 10^{23} \equiv 10^{24}$

(L) –19

(M) 5

(N) 7

TOPIC 2

SIGNIFICANT FIGURES

Significant figures are the trustworthy digits and the first uncertain digit in a measurement.

For instance, if the length measured is 264.5 cm, the digits 2, 6, and 4 are clear, whereas the number 5 is uncertain. As a result, there are four significant figures. Example: The period of oscillation of a pendulum is 1.62 s. Here 1 and 6 are reliable and 2 is uncertain. Thus, the measured value has three significant figures.

Rules for Determining Number of Significant Figures

(1) All non-zero digits are significant. 235.75 has five significant figures.

(2) All zeroes between two non-zero digits are significant. 2016.008 has seven significant figures.

(3) All zeroes occurring between the decimal point and the non-zero digits are not significant, provided there is only a zero to the left of the decimal point. 0.00652 has three significant figures.

(4) All zeroes written to the right of a non-zero digit in a number without a decimal point are not significant. This rule does not work if the zero is a result of measurement. 54000 has two significant figures. 54000 m has five significant figures.

(5) All zeroes occurring to the right of a non-zero digit in a number written with a decimal point are significant. 32.2000 has six significant figures. When a number is written in the exponential form, the exponential term does not contribute towards the significant figures. 2.465×10^5 has four significant figures.

Rules of Arithmetic Operations with Significant Figures

In arithmetical operations involving significant figures, the answer is reported in such a way that it reflects the reliability of the least precise operation.

Addition and Subtraction: For addition and subtraction, look at the decimal part (*i.e.*, to the right of the decimal point) of the numbers only and follow the steps as:

(1) Count the number of significant figures in the decimal portion of each number in the problem. (The digits to the left of the decimal place are not used to determine the number of decimal places in the final answer).

(2) Add or subtract in the normal way.

(3) Round off the answer to the least number of places in the decimal portion of any number in the problem.

Multiplication and Division: In multiplication or division the number of significant figures in the final answer is the same as the minimum number of the significant figures in the physical quantities being operated.

Exponential notation: Sometimes this would require expressing the answer in exponential notation.

Rules for Rounding off the uncertain digits:

(1) The preceding digit is raised by 1 if the uncertain digit to be dropped is more than 5 and is left unchanged if the latter is less than 5.

Example:

x = 5.686 is rounded off to 5.69 (as 6 > 5)

x = 3.462 is rounded off to 3.46 (as 2 < 5)

(2) If the uncertain digit to be dropped is 5, the preceding digit is raised by 1 if it is odd and is left unchanged if it is even.

Example: 7.735 when rounded off to three significant figures becomes 7.74 as the preceding digit is odd. 7.745 is rounded off to 7.74 as preceding digit is even.

Rules for Determining Uncertainty in Results of Arithmetic Calculations

To calculate the uncertainty, the below steps should be followed:

(1) Add the lowest amount of uncertainty in the original numbers.

Example: Uncertainty for 3.2 will be ± 0.1 and for 3.22 will be ± 0.01.

Calculate these in percentage also.

(2) After the calculations, the uncertainties get multiplied/divided/added/subtracted.

(3) Round off the decimal place in the uncertainty to get the final uncertainty result.

Example: For a rectangle, if length, l = 16.2 cm and breadth, b = 10.1 cm.

Then, take length, l = 16.2 ± 0.1 cm or 16.2 cm ± 0.6% and breadth, b = 10.1 ± 0.1 cm or 10.1 cm ± 1%.

On Multiplication,

Area = length × breadth = 163.62 cm^2 ± 1.6% and 163.62 ± 2.6 cm^2.

Therefore, after rounding off, area = 164 ± 3 cm^2. Hence, 3 cm^2 is the uncertainty or the error in estimation.

Rules

(1) For a set of experimental data of 'n' significant figures, the result will be valid to 'n' significant figures or less (only in case of subtraction).

Example: 12.9 – 7.06 = 5.84 or 5.8 (rounding off to lowest number of decimal places of original number).

(2) The relative error of a value of number specified to significant figures depends not only on n but also on the number itself.

Example: Accuracy for two numbers 1.02 and 9.89 is ± 0.01.

But relative errors will be:

For 1.02, $\left(\pm\frac{0.01}{1.02}\right) \times 100\% = \pm 0.1\%$

For 9.89, $\left(\pm\frac{0.01}{9.89}\right) \times 100\% = \pm 0.1\%$

Hence, the relative error depends upon the number itself.

(3) Intermediate results in multi-step computation should be calculated to one more significant figure in every measurement than the number of digits in the least precise measurement.

Example: $\frac{1}{9.58} = 0.1044$

Now, $\frac{1}{0.104} = 9.56$

and $\frac{1}{0.1044} = 9.58$

Hence, taking one extra digit gives more precise results and reduces rounding off errors.

Example 1.2: Write the following in scientific notation.

(A) 3256 g (B) 0.0010 g

(C) 50000 g (D) 0.3204

Ans. (A) 3.256×10^3 g (B) 1.0×10^{-3} g

(C) 5.0000×10^4 g (D) 3.204×10^{-1}

TOPIC 3

DIMENSIONS OF PHYSICAL QUANTITIES

Dimensions of the 7 fundamental quantities are – Length [L], Mass [M], Time [T], Electric current [A], Thermodynamic temperature [K], Luminous intensity [cd] and Amount of substance [mol].

Examples, Volume = Length × Breadth × Height

$= [L] \times [L] \times [L] = [L]^3 = [L^3]$

Force = Mass × Acceleration = $\frac{[M]}{[LT^{-2}]} = [MLT^{-2}]$

A quantity's other dimensions are always zero. For example, only length has three dimensions in volume, whereas mass, time, and other variables have none. The superscript 0 is used to denote zero dimensions. $[M^0]$. Dimensions do not take into account the magnitude of a quantity.

Dimensional Formula and Dimensional Equation

The Dimensional Formula is an expression that explains how and which basic quantities are used to indicate the dimensions of a physical quantity. An equation derived by equating a physical quantity with its dimensional formula is known as a dimensional equation.

Physical Quantity	Dimensional Formula	Dimensional Equation
Volume	$[M^0 L^3 T^0]$	$[V] = [M^0 L^3 T^0]$
Speed	$[M^0 L T^{-1}]$	$[v] = [M^0 L T^{-1}]$
Force	$[M L T^{-2}]$	$[F] = [M L T^{-2}]$
Mass Density	$[M L^{-3} T^0]$	$[\rho] = [M L^{-3} T^0]$

Dimensional Analysis

The only physical quantities that can be added and subtracted are those with the same dimensions. This is known as the principle of dimensional homogeneity. Dimensions can be multiplied and cancelled in the same way that they can in traditional algebraic approaches. In mathematical equations, the dimensions of the quantities on both sides must always be the same.

Special functions such as trigonometric, logarithmic, and ratio of equivalent physical values have dimensionless arguments. In the case of dimensionless quantities, equations are unreliable.

Example: Distance = Speed × Time;

In Dimension terms, $[L] = [LT^{-1}] \times [T]$

Since, dimensions can be cancelled like algebra, dimension [T] gets cancelled and the equation becomes [L] = [L].

Applications of Dimensional Analysis

Checking Dimensional Consistency of Equations

Dimensionally correct equation must have the same dimensions on both sides of the equation. A dimensionally accurate equation does not have to be correct; nevertheless, a dimensionally incorrect equation is always erroneous. It can check for dimensional validity, but it won't reveal the actual relationship between the physical quantities.

Example: $x = x_0 + v_0 + \left(\frac{1}{2}\right)at^2$

Or, Dimensionally, $[L] = [L] + [LT^{-1}] + [LT^{-2}][T^2]$

Where, x = Distance travelled in time t,

x_0 = starting position,

v_0 = initial velocity,

a = uniform acceleration.

Dimensions on both sides will be [L] and [T] not get cancelled out. Hence, this is a dimensionally incorrect equation.

Deducing Relation Among Physical Quantities

To deduce a relationship between physical quantities, we must first understand one quantity's dependency on others (or independent variables) and treat it as a product form of dependence. This approach cannot be used to obtain dimensional less constants.

Example: Let, the time period T of oscillations of a simple pendulum depends on its length (*l*), gravitational acceleration (*g*) and mass of the bob (*m*). Thus,

$$T = k\, l^x g^y m^z \quad \text{...(i)}$$

Or $[L^0M^0T^1] = [L^1]^x [L^1T^{-2}]^y [M^1]^z = [L^{x+y}T^{-2y} M^z]$

Means, $x + y = 0, -2y = 1$ and $z = 0$.

On solving above equation,

So, $x = \frac{1}{2}, y = -\frac{1}{2}$ and $z = 0$

So, from e.q. (i), we can write the desired relation as:

$$T = k\sqrt{\frac{l}{g}}$$

Conversion of System of Units

The method of dimensional analysis can be used to obtain the value of the physical quantity in some other system when its value in one system is given.

As discussed earlier,

the measurement of a physical quantity is given by,

$$X = nu$$

If the unit of one system is given as:

$$X_1 = n_1u_1$$

And the unit in other system is given as:

$$X_2 = n_2u_2$$

Then, according to principle of homogeneity,

$$X_1 = X_2$$

So, $n_1u_1 = n_2u_2$

$$n_2 = \frac{n_1u_1}{u_2} \quad \text{—(i)}$$

Where, u_1 and u_2 are the different units of same quantity and on the other hand n_1 and n_2 are their numerical values.

Let M_1, L_1 and T_1 be the fundamental units of mass, length and time in one system and M_2, L_2 and T_2 be the fundamental units of other system.

So, $u_1 = [M_1^aL_1^bT_1^c]$

and, $u_2 = [M_2^aL_2^bT_2^c]$

Then according to the equation (i),

$$n_2 = \frac{n_1\left(M_1^aL_1^bT_1^c\right)}{\left[M_2^aL_2^bT_2^c\right]}$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Limitations of Dimentional Analysis

(1) It doesn't provide any information about the physical quantity whether it is scalar or vector.

(2) It doesn't give any kind of information about dimensionless constant of any given formula.

(3) We can't derive any formula or relation, in which more than three quantities are related to each other.

(4) If any physical quantity has trigonometric function, logarithmic function or exponential function, which are dimensionless, then we can't derive relation among those quantities.

Example 1.3: The equation of the state of some gases can be expressed as $\left(P+\frac{A}{V^2}\right)(V - b) = RT$ (Van Der Waals Equation), where P is the pressure, V is the volume, T is the absolute temperature and A, b and R are constants. Find dimension of A.

Ans. According to the principle of homogeneity,

$\frac{A}{V^2}$ is dimensionally equal to P.

$$P = \frac{A}{V^2}$$

$$[M^1L^{-1}T^{-2}] = \frac{A}{[L^3]^2}$$

$$[M^1L^{-1}T^{-2}] = \frac{A}{[L]^6}$$

$$\Rightarrow \quad A = [ML^5T^{-2}]$$

Example 1.4: The dimensions of kinetic energy is:

(a) $[M^2L^2T]$ **(b) $[ML^2T]$**

(c) $[ML^2T^{-2}]$ **(d) $[ML^2T^{-1}]$**

Ans. (c) $[ML^2T^{-2}]$

Explanation: The formula for the kinetic energy is written as:

$$\text{Kinetic energy, K.E.} = \frac{1}{2}mv^2$$

$$= M\,[LT^{-1}]^2$$

$$= [ML^2\,T^{-2}]$$

Example 1.5: Case Based:

Every measurement involves errors. Thus, the result of measurement should be reported in a way that indicates the precision of measurement. Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus the first digit that is uncertain. The reliable digits plus the first uncertain digit are known as significant digits or significant figures. If we say the period of oscillation of a simple pendulum is 1.62 s, the digits 1 and 6 are reliable and certain, while the digit 2 is uncertain. Thus, the measured value has three significant figures.

(A) Give rules for addition and subtraction operations with significant figure.

(B) Give rules for multiplication and division operations with significant figure.

(C) Significant figures in 54200 cm are:

(a) 5 **(b) 4**

(c) 3 **(d) none of these**

(D) All the non-zero digits are:

(a) significant

(b) non significant

(c) data is insufficient

(d) none of these

(E) Assertion (A): The number of significant figures depends on the least count of measuring instrument.

Reason (R): Significant figures define the accuracy of measuring instrument.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

Ans. (A) In addition or subtraction, the final result after the operation should have as many decimal places as there are in the number with the least decimal places.

(B) In multiplication or division, the final result after operation should have as many significant figures as there are in the original number with the least significant figures.

(C) *(c) 3*

Explanation: According to the rules of significant figures.

The terminal or trailing zero(s) in a number without a decimal point are not significant.

Thus 54200 has only 3 significant figures.

(D) *(a) significant*

Explanation: As we know that the rules of significant figures, All the non-zero digits are significant.

(E) *(c) A is true but R is false.*

Explanation: Assertion is correct. The significant digits determine precision of the instrument, let's say using scale a value read is 1.8 m, using more precise instrument the reading can be 1.83.

Accuracy is closeness of a measured value to actual value, which is independent of measuring instrument least count.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. The numerical value of 1 J in the new system, in which length is measured in 10 meter, mass in 10 kg and time in 1 minute, will be:

(a) 36×10^{-4} (b) 36×10^{-3}

(c) 36×10^{-2} (d) 36×10^{-1}

Ans. *(d) 36×10^{-1}*

Explanation: $n_1 u_1 = n_2 u_2$

$$1 \times \left(\frac{\text{kg} \times \text{m}^2}{\text{s}^2}\right) = n_2 \left(\frac{10\text{ kg} \times (10\text{ m})^2}{\text{s}^2}\right)$$

$$n_2 = \frac{60 \times 60}{10 \times 100} = 3.6 \text{ unit}$$

Caution

➥ *Students should know that when dealing with basic units or fundamental units, always change minute to seconds. In an equation, multiplication of unity and multiplication of 60 can cause a major error in any calculation.*

2. The speed (v) of sound in a gas in given by

$$v = kP^x \rho^y$$

Where k is dimensionless constant, P is pressure, and ρ is the density,

(a) $x = \frac{1}{2}, y = \frac{1}{2}$ (b) $x = -\frac{1}{2}, y = -\frac{1}{2}$

(c) $x = \frac{1}{2}, y = -\frac{1}{2}$ (d) $x = -\frac{1}{2}, y = \frac{1}{2}$

[Delhi Gov. QB 2022]

Ans. *(c) $x = \frac{1}{2}, y = -\frac{1}{2}$*

Explanation: Let velocity be represented in terms of pressure and density as follows:

$\therefore v \propto p^x \times s^y$

Representing each of them in terms of basic physical quantities:

$\Rightarrow LT^{-1} \propto [ML^{-1}T^{-2}]^x \times [ML^{-3}]^y$

$\Rightarrow LT^{-1} \propto [M]^{x+y} \times [L]^{-x-3y} \times [T]^{-2x}$

Comparing exponents of similar terms:

$\therefore \quad -2x = -1$

$\Rightarrow \quad x = \frac{1}{2}$

Again,

$\therefore \quad -x - 3y = 1$

$\Rightarrow \quad y = -\frac{1}{2}$

So, final answer is:

$$x = \frac{1}{2} \text{ and } y = -\frac{1}{2}$$

3. A physical quantity is measured and the result is expressed as nu where, u is the unit used and n is the numerical value. If the result is expressed in various units, then

a) $n \propto$ size of u (b) $n \propto u^2$

(c) $n \propto \sqrt{u}$ (d) $n \propto \frac{1}{u}$

Ans. *(d)* $n \propto \frac{1}{u}$

Explanation: If the unit is increasing then the numerical value is decreasing and if the unit is decreasing then the numerical value is increasing.

Hence, $n \propto \frac{1}{u}$ is the correct option.

Caution

Students must know that if the number of observations is made n times, then the random error reduces to $\left(\frac{1}{n}\right)$ times.

4. Which one of the following pair of quantities has the same dimensions?

(a) Force and work done

(b) Momentum and impulse

(c) Pressure and force

(d) Time period and frequency

[Delhi Gov. SQP 2022]

Ans. *(b) Momentum and impulse*

Explanation: Impulse is defined as the change in momentum.

Impulse, $I = \Delta p = mv - mu$

$= [MLT^{-1}]$

Hence, both Impulse and momentum have the same units/dimensions.

5. Which one of the following quantities has dimensions different from the remaining three?

(a) Energy per unit volume

(b) Force per unit area

(c) Product of voltage and charge per unit volume

(d) Angular momentum per unit mass

Ans. *(d) Angular momentum per unit mass*

Explanation: Energy per unit volume,

$$F = \frac{\text{Energy}}{\text{Volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$$

$$= [ML^{-1}T^{-2}]$$

$$\text{Force per unit area} = \frac{[MLT^{-2}]}{[L^2]}$$

$$= [ML^{-1}T^{-2}]$$

Product of the charge per unit volume and voltage

$$= Q\frac{(IT)V}{\text{volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$$

$$= [ML^{-1}T^{-2}]$$

Angular momentum per unit mass

$$= \frac{[M^1L^2T^{-1}]}{[M]}$$

$$= [L^2T^{-1}]$$

Caution

Students should know that different quantities can have the same dimensional unit. Many students on finding the same dimension of a quantity as of some other quantity cancel out the process and redo the question.

6. The dimension of light year is ....

(a) T (b) LT^{-1}

(c) L (d) T^{-1}

[Delhi Gov. SQP 2022]

Ans. *(c) L*

Explanation: We know that, Light year is the unit of distance. The dimension of distance is [L].

So, the dimensional formula of Light year is [L].

7. Find the density of the material of the body in correct significant figures if the mass and volume of a body are 4.237 g and 2.5 cm^3 respectively.

(a) 1.6048 g cm^{-3} (b) 1.69 g cm^{-3}

(c) 1.7 g cm^{-3} (d) 1.695 g cm^{-3}

Ans. *(c) 1.7 g cm^{-3}*

Explanation: The density of a material is given by, $\rho = \frac{m}{V}$.

Where, m is mass and v is volume.

$$\rho = \frac{4.237}{2.5}$$

$$= 1.6948 \text{ g/cm}^3.$$

We know that in multiplication or division, the final answer should have as many significant figures as in given data with a minimum number of significant figures. Here, 2.5 cm^3 have the minimum number of significant figures equal to two. Therefore, the final answer should have two significant figures. On rounding off 1.6948 g/cm^3 to two significant figures, we get, 1.7 g/cm^3.

8. If momentum (P), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula:

(a) $[P^1 A^{-1} T^1]$ (b) $[P^2 A^1 T^1]$

(c) $[P^1 A^{-1/2} T^1]$ (d) $[P^1 A^{1/2} T^{-1}]$

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Ans. *(d) $[P^1A^{1/2}T^{-1}]$*

Explanation: Let energy,

$$E \propto P^a A^b T^c$$

Or $$E = k P^a A^b T^c$$

Or $$[ML^2T^{-2}] = [ML^1T^{-1}]^a [L^2]^b [T]^c$$

$$= [M^a L^{a+2b} T^{-a+c}]$$

Hence, $a = 1$, $b = 12$, $c = -c$

Dimensional formula for E is $[P^1 A^{1/2} T^{-1}]$.

9. **The dimensional unit of permeability is:**

(a) $[M^{-1} LT^{-2} A]$ (b) $[ML^{-2} T^{-2}A^{-1}]$

(c) $[MLT^{-2} A^{-2}]$ (d) $[MLT^{-1} A^{-1}]$

Ans. *(c)* $[MLT^{-2} A^{-2}]$

Explanation: Units of permeability are equivalent to $\frac{N}{Amp^2}$

Thus, dimensions are $= \frac{M^1L^1T^{-2}}{A^2}$

$= [M^1LT^{-2}A^{-2}]$

10. **The dimension of $\frac{1}{2}\varepsilon_0 E^2$, where, ε_0 is permittivity of free space and E is electric field, is:**

(a) $[MLT^{-1}]$ (b) $[ML^2T^{-2}]$

(c) $[ML^{-1}L^{-2}]$ (d) $[ML^2T^{-1}]$

Ans. *(c)* $[ML^{-1}L^{-2}]$

Explanation: $\frac{1}{2}\varepsilon_0 E^2$ = energy density

$= \frac{\text{Energy}}{\text{Volume}} = [M^1L^{-1}T^{-2}]$.

$[MLT^{-1}]$ is the dimensional unit of impulse. $[ML^2T^{-1}]$ is the dimensional unit of angular momentum and heat and $[ML^2T^{-1}]$ is the dimensional unit of Planck's constant and angular impulse.

11. **The number of significant figures in 0.06900 ....**

(a) 2 (b) 4

(c) 6 (d) 5

[Delhi Gov. SQP 2022]

Ans. *(b)* 4

Explanation: As we know, the trailing zero(s) in a number with a decimal point are significant.

So, 0.06900 has FOUR significant figures.

12. **The atmospheric pressure is 10^6 dyne/cm^2 in CGS system. What is its value in SI unit?**

(a) 10^5 Newton/m^2 (b) 10^6 Newton/m^2

(c) 10^4 Newton/m^2 (d) 10^3 Newton/m^2

Ans. *(a)* 10^5 newton/m^2

Explanation: 1 dyne = 1 g × 1 cm/s^2

1 dyne = 10^{-3} kg × 10^{-2} m/s^2

[∵ 1 m = 100 cm
And 1 kg = 1000 g]

1 dyne = 10^{-5} N

$$10^6 \text{ dyne/cm}^2 = \frac{10^6 \times 10^{-5} N}{(10^{-2})^2 m^2}$$

$$= 10^5 \text{ N/m}^2$$

13. **The dimensions of kinetic energy is same as that of:**

(a) force (b) pressure

(c) work (d) momentum

Ans. *(c)* *work*

Explanation:

According to Work – Energy Theorem,

Work = ΔK.E.

Hence, their dimensions will also be equal.

14. **Out of 4.0 and 4.00, which is more accurate?**

(a) 4.0

(b) 4.00

(c) Both are equally accurate

(d) Nothing can be said

Ans. *(b)* 4.00

Explanation: The two quantities appear to be identical, but they are not. 4.0 g has two significant digits and so has a precision of 0.1 part in 4. Because 4.00 g has three significant numbers, its accuracy is 0.01 parts in 4. As a result, 4.00 g is more exact than 4.0 g.

Assertion-Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

15. **Assertion (A):** Force and pressure cannot be added.

Reason (R): The dimensions of force and pressure are different.

[Delhi Gov. QB 2022]

Ans. *(a)* *Both A and R are true and R is the correct explanation of A.*

Explanation: Two quantities with different dimensions cannot be added due to dimensional consistency. Since force and pressure (Force per unit area) have distinct dimensions, they cannot be combined.

16. **Assertion (A):** Dimensional constants are the quantities whose values are constant.

Reason (R): Dimensional constants are dimensionless.

Ans. *(c)* *A is true but R is false.*

Explanation: Dimensional constants are the quantities whose value are constant and they posses dimension for example velocity of light in vacuum, universal gravitational constant, boltsman, plank constant, etc.

17. Assertion (A): **When we change the unit of measurement of a quantity, its numerical value changes.**

Reason (R): **Smaller the unit of measurement smaller is its numerical value.**

Ans. *(c) A is true but R is false.*

Explanation: Assertion is correct that changing the unit of measurement changes the numerical value of the quantity.

For example:

Let the mass of the body to be 1 kg.

Its mass in cgs unit,

$$m = 1 \text{ kg} \times \frac{1000 \text{ g}}{\text{kg}}$$

$$= 1000 \text{ g}$$

Hence, the numerical value of the mass gets changed.

Also, we see that smaller the unit of measurement, greater is its numerical value.

18. Assertion (A): **Number of significant figure in 0.005 is one and in 0.500 is three.**

Reason (R): **This is because zeros are not significant.**

[Delhi Gov. QB 2022]

Ans. *(c) A is true but R is false.*

Explanation: A zero between the decimal point and the first non-zero digit in a value smaller than one is not relevant. However, the zeros to the right of the final non-zero digit are essential.

19. Assertion (A): **Surface energy of a liquid is numerically equal to its surface tension.**

Reason (R): **The dimensional formula of surface energy and surface tension is $[ML^0T^{-2}]$.**

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: The potential energy per unit area of the surface film is called the surface energy.

Surface tension = Surface energy

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}} = \frac{[MLT^{-2}]}{[L]}$$

$$= [ML^0T^{-2}]$$

$$\text{Surface energy} = \frac{\text{Energy}}{\text{Area}} = \frac{[ML^2T^{-2}]}{[L^2]}$$

$$= [ML^0T^{-2}]$$

Thus, both Assertion and Reason are true, but reason is not a correct explanation for the assertion.

20. Assertion (A): **The measure of physical quantity is independent of the system of units.**

Reason (R): **The smaller is unit, the bigger is the measure of the physical quantity and vice-versa.**

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: The measure of the physical quantity is given by X = *nu*, where *n* is the size of the unit and *n* is the numerical value of the physical quantity X for the selected unit. It follows that if the size of the chosen unit is small, then the numerical value of the quantity will be large and vice-versa. Thus, both assertion-reasons are correct.

21. Assertion (A): **The number of significant figures in 0.100 is 1.**

Reason (R): **The zeros at the end of a number are always meaningless.**

Ans. *(d) A is false and R is also false.*

Explanation: All zeros to the right of the last non-zero digit after the decimal point are significant. Therefore, the number of significant figures is 0.100 is 3.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

22. *Measurement is one subject matter which isn't always best taught in math however additionally in Physics and Chemistry due to the fact that each problem calls for the information of size, so that it will degree the quantities. Measuring a amount does now no longer continually provide a superbly correct answer. Only Ideal measuring gadgets can offer a superbly correct answer. Practically size results in elements of a solution known as dependable digits and unsure digits. The reliability of a size is indicated via way of means of the variety of digits used to symbolize it. To specific it extra correctly we* *specific it with digits which are recognized with certainty. These are known as large figures. They incorporate all of the positive digits plus one dubious digit in a variety.*

(A) Give any three rules of significant figures.

(B) Find the total number of significant digits in 5400.

(C) What is the rounded off value of 1.6932?

Ans. (A) (1) All non-zero digits are significant.

(2) All zeroes between two non-zero digits are significant.

(3) All zeroes occurring between the decimal point and the non-zero digits are not significant, provided there is only a zero to the left of the decimal point.

(B) There are 2 significant figure in 5400 because as we know if there is zero at the end of any number without decimal point, should be considered as non-significant figure.

(C) The rounding off value of given no. 1.7 because according to the rule of rounding off if the digit to be dropped is more than 5, then the preceding digit is raised by 1.

23. *A mathematical calculation can't increase the precision of a physical measurement therefore the number of significant figures in the sum or product of a group of measurement cannot be greater than minimum number of significant figures.* **[Delhi Gov. QB 2022]**

(A) A car runs 1200 m in 22.5 sec. The average speed of a car in appropriate significant figures.

(a) 53.3 m/s (b) 53.33 m/s

(c) 53.333 m/s (d) None of these

(B) The radius of a uniform wire $r = 0.024$ cm. Take $\pi = 3.142$, then area of cross-section upto appropriate significant figures.

(a) 0.001808 cm^2 (b) 0.0018086 cm^2

(c) 0.0018 cm^2 (d) 18.08 cm^2

(C) The volume of sphere is 2.42 cm^3. The volume of 12 spheres taking into account the significant figures.

(a) 29.0 cm^3 (b) 29.04 cm^3

(c) 29.1 cm^3 (d) 29 cm^3

(D) The length of a rectangular block is 2.5 m, breadth is 1.75 m. The area of surface of block taking into account of the significant figures.

(a) 4.38 cm^2 (b) 4.3 cm^2

(c) 4.4 cm^2 (d) 4.375 cm^2

(E) The number of significant figures in 0.06900 is ....

(a) 5 (b) 4

(c) 2 (d) 3

Ans. (A) *(a) 53.3 m/s*

Explanation: Given that

Distance = 1200 m

Time = 22.5 sec

Average speed = ?

We know that

$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time Taken}}$$

Hence,

$$S = \frac{1200}{22.5}$$

$$= 53.333333 \text{ m/s}$$

In significant figure, the average speed is 53.3 m/s.

(B) *(c) 0.0018 cm^2*

Explanation: According to the question:

$r = 0.024$ cm

$\pi = 3.142$

As we know that,

Area of Cross-Section of wire = πr^2

$$A = 3.142 \times 0.024 \times 0.024$$

$$= 0.001809792 \text{ cm}^2$$

So, the value of the area of cross-section in significant figure is 0.0018 cm^2.

(C) *(d) 29 cm^3*

Explanation: Given that

Volume of sphere = 2.42 cm^3

There are 12 spheres.

So, Total volume = 12×2.42

$= 29.04$ cm^3

Hence, the value of the total volume in significant figures is 29 cm^3.

(D) *(a) 4.38 cm^2*

Explanation: Given that:

Length of block, $l = 2.5$ m

Breadth of block, $b = 1.75$ m

As we know that,

Area of rectangular block = $l \times b$

$= 2.5 \times 1.75$

$= 4.375$ cm^2

Hence, the value of the area in significant figures is 4.38 cm^2.

(E) *(b) 4*

Explanation: According to the rule of significant figure, all zeroes occurring between the decimal point and the non-zero digits are not significant, provided there is only a zero to the left of the decimal point. So, the value 0.06900 has 4 significant figures.

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

24. What is the difference between mN, Nm and nm? **[Delhi Gov. QB 2022]**

Ans. nm stands for nanometre. 1 nm = 10^{-9}m

mN stands for milli newton, 1mN = 10^{-3} N

Nm stands for Newton metre, which is a unit of work.

25. Why don't we need a very large number of units for the measurement even though the number of units which we measure is very large?

Ans. This is possible because the various physical quantities are related to each other and so their units can be expressed in seven basic or fundamental units. That's why we don't require a very large number of units to measure the value.

26. How can we check the dimensional correctness of a physical equation?

Ans. To check the dimensional correctness of a physical equation, we make use of the principle of homogeneity of dimensions. If the dimensions of all the terms of both sides of the equation are same, then the equation is dimensionally correct.

27. Name the physical quantities that have dimensional formula [$ML^{-1}T^{-2}$]. **[Delhi Gov. QB 2022]**

Ans. Stress, pressure, modulus of elasticity.

28. How many kg make 1 unified atomic mass unit?

Ans.

$$1 \text{ amu} = \frac{1}{12} \text{ mass of 1 atm of } ^{12}C$$

$$= \frac{1}{12} \times \frac{12}{6.022 \times 10^{23}} g$$

$$= 1.66 \times 10^{-24} g$$

$$1 \text{ kg} = 1000 \text{ g}$$

$$x \text{ kg} = 1.66 \times 10^{-24} g$$

$$x = \frac{1.66 \times 10^{-24}}{1000}$$

$$= 1.66 \times 10^{-27} \text{ kg}$$

29. What are the characteristics of a physical unit/standard?

Ans. Characteristics of a physical unit are that it should be well-defined and should be of convenient size, *i.e.*, neither too small nor too large in comparison with the measurable physical quantity. It should not change with time and easily reproducible, imperishable or indestructible and must be internationally acceptable and easily accessible.

30. Name physical quantities whose units are electron volt and Pascal.

Ans. Energy and pressure. Pressure is defined as force per unit area. It is usually more convenient to use pressure rather than force to describe the influences upon fluid behavior. The standard unit for pressure is the Pascal, which is a Newton per square meter. Electron volt, unit of energy commonly used in atomic and nuclear physics, equal to the energy gained by an electron (a charged particle carrying unit electronic charge) when the electrical potential at the electron increases by one volt.

31. The radius of the atom is of the order of 1 Å and the radius of the nucleus is of the order of fermi. How many magnitudes higher is the volume of atoms as compared to the volume of the nucleus?

Ans. According to the question,

Radius of atom 1 Å = 10^{-10} m

Radius of nucleus ≈ fermi = 10^{-15}m

Volume of atom,

$$V_{atom} = \frac{3}{4}\pi R_A^3$$

Volume of nucleus,

$$V_{nucleus} = \frac{3}{4}\pi R_N^3$$

$$\frac{V_{atom}}{V_{nucleus}} = \frac{\frac{4}{3}\pi R_A^3 - A}{\frac{4}{3}\pi R_N^3}$$

$$= \left(\frac{10^{-10}}{10^{-15}}\right)^3$$

$$= 10^{15}$$

32. If the velocity of light *c*, acceleration due to gravity *g* and atmospheric pressure P are the fundamental quantities, find the dimensions of length.

Ans. Let $c^x g^y P^z$ be dimensions of length.

$$[M^0LT^{-1}]^x [M^0LT^{-2}]^y [ML^{-1}T^{-2}]^z = M^0LT^0$$

⇒ Comparing powers of M, L and T.

We get, $z = 0$;

$$x + y = 1;$$

$$-x - 2y = 0$$

$$-2y + y = 1$$

⇒ $x = -2y$

⇒ $y = -1$

⇒ $x = 2$

$\therefore$ Dimensions (length) $= c^x g^y P^z$

$$= c^2 g^{-1} P^0 = \frac{c^2}{g}.$$

33. Name the technique used in locating.

(A) an under water obstacle

(B) position of an aeroplane in space.

[Delhi Gov. QB 2022]

Ans. (A) SONAR → Sound Navigation and Ranging.

(B) RADAR →Radio Detection and Ranging.

34. Comment on the statement: "To define a physical quantity for which no method of measurement is given or known has no meaning."

Ans. The given statement is not correct. A physical quantity, if it is called so, must have a physical meaning. If it cannot be measured by any direct method, there must be some indirect method for its measurement. Entropy is one such physical quantity.

35. If the units of force and length each are doubled, then how many times would the unit of energy be affected?

Ans. Energy = Work done = Force × length

So, when the units of force and length each are doubled, the unit of energy will increase four times.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

36. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

Ans. A LASER is a source of a very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. Time taken by the laser beam to return to Earth after reflection from the Moon $= 2.56$ s

Speed of light $= 3 \times 10^8$ m/s

Time taken by the laser beam to reach Moon

$$= \frac{1}{2} \times 2.56 = 1.28 \text{ s}$$

Radius of the lunar orbit = Distance between the Earth and the Moon

$$= 1.28 \times 3 \times 10^8$$
$$= 3.84 \times 10^8 \text{m}$$
$$= 3.84 \times 10^5 \text{ km}$$

37. If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s. What is the unit of Mass in this system of units?

[Delhi Gov. QB 2022]

Ans. First write the dimension of each quantity and then relate them.

Force $[F] = [MLT^{-2}] = 100$ N ...(i)

Length $[L] = [L] = 10$ m ...(ii)

Time $[t] = [T] = 100$ s ...(iii)

Substituting values of L and T form Eqn. (ii) and (iii) in Eqn. (i), we get

$$M \times 10 \times (100)^{-2} = 100$$

$$= \frac{M \times 10}{100 \times 100} = 100$$

$$M = 100 \times 1000 \text{ kg}$$
$$= 10^5 \text{ kg}$$

38. If the centripetal force is of the form $m^a v^b r^c$, find the values of a, b and c.

Ans. Dimensionally,

Force = (Mass)^a × (velocity)^b × (length)^c

$$[MLT^{-2}] = [M^a L^b T^{-c}] = [M^a L^b T^{-c}]$$

Equating the exponents of similar quantities,

$$a = 1, b + c = 1, -b = -2$$

or, $a = 1, b = 2, c = -1$

or, $F = \frac{mv^2}{r}$

39. Magnitude of force experienced by an object moving with speed v is given by $F = kv^2$. Find dimensions of k. [Delhi Gov. QB 2022]

Ans. Given relation is, $F = Kv^2$

Taking dimensions of each term, we get

$$[MLT^{-2}] = [K][LT^{-1}]^2$$

$$\Rightarrow \quad [K] = [MLT^{-2}][L^2T^{-2}]$$

$$[K] = [ML^{-1}]$$

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

40. A calorie is a unit of heat or energy and it equals 4.2 J where 1 J = 1 kg m^2s^{-2}. Suppose we employ a system of units in which unit of mass is α kg, unit of length is β m, unit of time γ s. What will be magnitude of calorie in terms of this new system?

[Delhi Gov. QB 2022]

Ans. We know that

$$n_1u_1 = n_2u_2$$

$$n_2 = n_1\frac{u_1}{u_2} = n_1\frac{[M_1^aL_1^bT_1^c]}{[M_2^aL_2^bT_2^c]}$$

$$= n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

SI System New system

$$n_1 = 4.2, n_2 = ?$$

$$M_1 = 1 \text{ kg}, M_2 = \alpha \text{ kg}$$

$$L_1 = 1 \text{ m}, L_2 = \beta \text{ m}$$

$$T_1 = 1 \text{ s}, T_2 = \gamma \text{ s}$$

$$1 \text{ cal} = 4.2 \text{ J} = 4.2 \text{ kg m}^2 \text{ s}^{-2}$$

$$a = 1, b = 2, c = -2$$

$$n_2 = 4.2\left[\frac{1 \text{ kg}}{\alpha \text{ kg}}\right]^1\left[\frac{1 \text{ m}}{\beta \text{m}}\right]^2\left[\frac{1 \text{ s}}{\gamma \text{s}}\right]^{-2}$$

$\therefore$ $$n^2 = 4.2\,\alpha^{-1}\beta^{-2}\gamma^2$$

$\therefore$ 1 cal = 4.2 $\alpha^{-1}\beta^{-2}\gamma^2$ in new system

41. If the velocity of light c, the constant of gravitation G and Planck's constant h be chosen as fundamental units, find the dimensions of mass, length and time in terms of c, G and h.

Ans. $[c] = LT^{-1}$, $[G] = M^{-1}L^3T^{-2}$, $[h] = ML^2T^{-1}$

$$\frac{[h][c]}{[g]} = \frac{ML^2T^{-1}LT^{-1}}{M^{-1}L^3T^{-2}} = M^2$$

Hence, $$[M] = h^{\frac{1}{2}}c^{\frac{1}{2}}G^{-\frac{1}{2}}$$

Again, $$\frac{[h]}{[c]} = \frac{ML^2T^{-1}}{LT^{-1}} = ML$$

$$[L] = \frac{h}{c[M]}$$

$$= \frac{h}{ch^{\frac{1}{2}}c^{\frac{1}{2}}G^{-\frac{1}{2}}}$$

$$= h^{-\frac{1}{2}}c^{-\frac{3}{2}}G^{-\frac{1}{2}}$$

As $$[c] = LT^{-1}$$

$$[T] = \frac{[L]}{c} = \frac{h^{\frac{1}{2}}c^{-\frac{3}{2}}G^{\frac{1}{2}}}{c} = h^{\frac{1}{2}}c^{-\frac{5}{2}}G^{\frac{1}{2}}$$

42. Deduce the dimensional formula for the following quantities:

(A) Gravitational constant

(B) Young's modulus

(C) Coefficient of viscosity.

[Delhi Gov. QB 2022]

Ans. (A) Universal constant of gravitation G.

$$\text{Gravitational Force} = G\frac{m_1m_2}{r^2}$$

$$\Rightarrow \quad G = \frac{F \cdot r^2}{m_1m_2}$$

$$[G] = \frac{[MLT^{-2}][L^2]}{[M][M]}$$

$$[G] = M^{-1}L^3T^{-2}$$

(B) Unit of Young's Modulus = N/m^2

$$= \text{kgm/s}^2\text{m}^2 = \frac{\text{kg}}{\text{ms}^2} = ML^{-1}T^{-2}$$

(C) Coefficient of viscosity

$$n = \frac{F}{A(dv/dx)}$$

$$\Rightarrow \quad = \frac{[MLT^{-2}]}{[L^2]\left[\frac{LT^{-1}}{L}\right]}$$

$$[n] = ML^{-1}T^{-1}$$

43. If the dimension of a physical quantity are given by $M^a L^b T^c$, then the physical quantity will be:

(A) Force, if $a = 0$, $b = -1$, $c = -2$

(B) Pressure, if $a = 1$, $b = -1$, $c = -2$

(C) Velocity, if $a = 1$, $b = 0$, $c = -1$

(D) Acceleration, if $a = 1$, $b = 1$, $c = -2$

Ans. (A) [Force] = $[MLT^{-2}]$

(B) [Pressure] = $[ML^{-1}T^{-2}]$

(C) [Velocity] = $[LT^{-1}]$

(D) [Acceleration] = $[LT^{-2}]$

44. The frequency of vibration of a string depends of on, (i) tension in the string (ii) mass per unit length of string, (iii) vibrating length of the string. Establish dimensionally the relation for frequency.

[Delhi Gov. QB 2022]

Ans. Expression of frequency is given as follows:

Frequency,

$$f \propto [L^a] \quad ...(i)$$

$$f \propto [F^b] \quad ...(ii)$$

$$f \propto [M^c] \quad ...(iii)$$

Combining eqn (i), (ii) and (iii) we can say:

$$f = [KL^aF^bM^c]$$

where $M = \frac{\text{Mass}}{\text{Unit length}}$

L = Length

F = Tension (Force)

Dimension of $f = [T^{-1}]$

Dimension of right side:

Dimension of force,

$$F = [MLT^{-2}]^b = [M^bL^bT^{-2b}]$$

Dimension of mass per unit length

$$= [ML^{-1}]^c$$

$$= [M^cL^{-c}]$$

So, $[T^{-1}] = [L^a][M^bL^bT^{-2b}][M^cL^{-c}]$

$$[M^0L^0T^{-1}] = M^{b+c}L^{a+b-c}T^{-2b}$$

Equating the dimensions of both sides, we get

$$b + c = 0, b + c = 0 \quad ...(i)$$

$$-c + a + b = 0 - c + a + b = 0 \quad ...(ii)$$

$$-2b = -1 - 2b = -1 \quad ...(iii)$$

On Solving the equations, we get

$$a = -1, b = \frac{1}{2} \text{ and } c = \frac{-1}{2}$$

$$\therefore \quad f = KL^{-1}F^{\frac{1}{2}}M^{\frac{-1}{2}}$$

Hence, expression of frequency will be as follows:

$$f = \frac{K}{L}\sqrt{\frac{F}{M}}$$

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

45. A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c, e, mass of electron, mass of proton) and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (-15 billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants?

Ans. The values of different fundamental constants are as follows:

Charge on an electron,

$$e = 1.6 \times 10^{-19}\ C$$

Mass of an electron,

$$m_e = 9.1 \times 10^{-31}\ kg$$

Mass of a proton,

$$m_p = 1.67 \times 10^{-27}\ kg$$

Speed of light,

$$c = 3 \times 10^8\ m/s$$

Gravitational constant,

$$G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2\ C^{-2}$$

We have to try to make permutations and combinations of the universal constants and see if there can be any such combination whose dimensions come out to be the dimensions of time. One such combination is:

$$\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \frac{1}{m_p m_e^2 c^3 G}$$

According to Coulomb's law of electrostatics,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{r^2}$$

or, $$\frac{1}{4\pi\varepsilon_0} = \frac{Fr^2}{e^2}$$

or $$\left(\frac{1}{4\pi\varepsilon_0}\right)^2 = \frac{F^2 r^4}{e^4}$$

According to Newton's law of gravitation,

$$F = G\frac{m_1 m_2}{r^2}$$

or $$G = \frac{Fr^2}{m_1 m_2}$$

Now, $$\left[\frac{e^4}{(4\pi\varepsilon_0)^2 m_p m_e^2 c^3 G}\right]$$

$$= \left[e^4\left(\frac{F^2r^4}{e^4}\right)\frac{1}{m_p m_e^2 c^3}\frac{m_1 m_2}{Fr^2}\right]$$

$$= \left[\frac{Fr^2}{mc^3}\right]$$

$$= \left[\frac{MLT^{-2}L^2}{ML^3T^{-3}}\right] = [T]$$

Clearly, the quantity under discussion has the dimensions of time. Substituting values in the quantity under discussion, we get

$$= \frac{(1.6\times10^{-19})^4(9\times10^9)^2}{(1.69\times10^{-27})(9.1\times10^{-31})^2(3\times10^8)^3(6.67\times10^{-11})}$$

$$= 2.1 \times 10^{16} \text{ second}$$

$$= \frac{2.1\times10^{16}}{60\times60\times24\times365.25} \text{ years}$$

$$= 6.65 \times 10^8 \text{ years}$$

$$= 10^9 \text{ years}$$

The estimated time is nearly one billion years.

46. (A) **Let us consider an equation $\frac{1}{2}mv^2 = mgh$ where m is the mass of the body, v its velocity, g is the acceleration due to gravity and h is the height. Check whether this equation is dimensionally correct.**

(B) **Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that period of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using the method of dimensions. [Delhi Gov. SQP 2022]**

Ans. (A) According to the Principle of Homogeneity, the dimensions of each term in a dimensional equation on both sides should be the same. Constant term are ignored.

So, according to the question,

$$mv^2 = mgh$$

On applying the units on both sides,

$$Kg.\,(m.sec^{-1})^2 = Kg.\,(m.sec^{-2}).m$$

$$Kg.m^2sec^{-2} = Kg.m^2sec^{-2}$$

Now applying dimensions on both sides,

$$[M^1][L^2][T^{-2}] = [M^1][L^2][T^{-2}]$$

Hence, We can say that given relation is dimensionally correct.

(B) The dependence of time period T on the quantities l, g and mass a product may be written as $T = kl^x g^y m^z$

where, k is dimensionless constant as x, y and z are the exponents. Taking dimensions on both sides, we have

$$[L^0M^0T^{-1}] = [L^1]^x[T^{-2}]^y[M^1]^z$$

$$[L^0M^0T^1] = M^zL^{x+y}T^{-2y}$$

On equating the dimensions on both sides, we have

$$x + y = 0$$

$$-2y = 1$$

$$\Rightarrow \quad y = -\frac{1}{2} \text{ and } x = \frac{1}{2}$$

and $z = 0$

So, that $$T = kl^{\frac{1}{2}}g^{\frac{-1}{2}}$$

or $$T = k\sqrt{\frac{l}{g}}$$

The value of constant k cannot be obtained by the method of dimensions. Here, it does not matter if some number multiplies the right side of this formula, because that does not affect its dimensions.

Actually, $k = 2\pi$ so that

$$T = 2\pi\sqrt{\frac{l}{g}}$$

47. **The time of oscillation T of a small drop of a liquid under surface tension (whose**

dimensions are those of force per unit length) depends upon the density d, the radius r and the surface tension σ. Derive the relation among the all quantities.

Ans. Given that:

Time of oscillation = T

Density = d

Radius = r

Surface tension = σ

According to question,

$$T \propto d^a r^b \sigma^c$$

$$T = K d^a r^b \sigma^c \quad \text{...(i)}$$

Where, K is the proportionality constant.

We know that, the dimensions of:

Time (T) = [T]

Density (d) = $[ML^{-3}]$

Radius (r) = [L]

Surface tension (σ) = $[MT^{-2}]$

From eqn. (i)...

$$[M^0L^0T^1] = K[M^1L^{-3}T^0]^a [M^0L^1T^0]^b [M^1L^0T^{-2}]^c$$

$$[M^0L^0T^1] = K[M]^{a+c}[L]^{-3a+b}[T]^{-2c}$$

On comparing both sides:

$$a + c = 0$$

$$-3a + b = 0$$

$$-2c = 1$$

After solving the equations, we get,

$$a = \frac{1}{2}$$

$$b = \frac{3}{2}$$

$$c = -\frac{1}{2}$$

Applying the value of a, b and c in eqn. (i),

$$T = K d^{\frac{1}{2}} r^{\frac{3}{2}} \sigma^{-\frac{1}{2}}$$

Hence,

$$T = K\sqrt{\frac{dr^3}{\sigma}}$$

NUMERICAL Type Questions

48. When 1 m, 1 kg and 1 min are taken as the fundamental units, the magnitude of the force is 36 units. What will be the value of this force in CGS system? (3m)

Ans. As, dimensional formula of force,

$F = [MLT^{-2}]$

n_1 = 36,

M_1 = 1 kg,

L_1 = 1 m,

T_1 = 1 min

= 60 s

n_2 = ?,

M_2 = 1 g,

L_2 = 1 cm,

T_2 = 1 s

So, conversion of 36 units into CGS system,

i.e.,
$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$n_2 = n_1\left[\frac{1\,\text{kg}}{1\,\text{g}}\right]^1\left[\frac{1\,\text{m}}{1\,\text{cm}}\right]^1\left[\frac{1\,\text{min}}{1\,\text{s}}\right]^{-2}$$

$$= 36\left[\frac{1000\,\text{g}}{1\,\text{g}}\right]\left[\frac{100\,\text{cm}}{1\,\text{cm}}\right]^1\left[\frac{60\,\text{s}}{1\,\text{s}}\right]^{-2}$$

$$= 10^3 \text{ dyne}$$

49. The escape velocity v of a body depends on: (i) the acceleration due to gravity 'g' of the planet. (ii) the radius R of the planet. Establish dimensionally the relation for the escape velocity. [Delhi Gov. QB 2022](3m)

Ans. We know that

Escape velocity, $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{gR}$

Hence, we can write

$$[v] = [g]^a[R]^b$$

$$[L^1T^{-1}] = [L^1T^{-2}]^a[L^1]^b$$

$$[L^1T^{-1}] = [L^{a+b}][T^{-2a}]$$

Comparing the power, we get

$$a + b = 1 \text{ and } -2a = -1$$

$$\Rightarrow \quad a = \frac{1}{2} \text{ and } b = \frac{1}{2}$$

$$\Rightarrow \quad v_o = \sqrt{gR}$$

50. Find an expression for viscous force F acting on a tiny steel ball of radius r moving in a viscous liquid of viscosity η with a constant speed v by the method of dimensional analysis. (3m)

Ans. It is given that viscous force F depends on (i) radius r of steel ball, (ii) coefficient of viscosity η of viscous liquid (iii) Speed v of the ball

i.e., $F = kr^a \eta^b v^c$,

where, k is dimensionless constant.

We know that,

Dimensional formula of :

$$\text{Force (F)} = [MLT^{-2}],$$

$$\text{Radius } (r) = [L]$$

$$\text{Viscocity } (\eta) = [M^1 L^{-1} T^{-1}]$$

and $\text{Speed } (v) = [LT^{-1}]$,

we have, $[MLT^{-2}] = [L]^a [M^1L^{-1}T^{-1}]^b [LT^{-1}]^c$

$$= [M^a L^{a-b+c} T^{-b-c}]$$

Comparing powers of M, L and T on either side of equation, we get

$$a = 1$$

$$a - b + c = 1$$

$$-b - c = -2$$

On solving, these above equations, we get

$$a = 1,$$

$$b = 1$$

and $c = 1$

Hence, the relation becomes,

$$F = kr\eta v$$